A new analysis of ^{14}O beta decay: branching ratios and CVC consistency

I.S. Towner* and J.C. Hardy
Cyclotron Institute, Texas A & M University, College Station, Texas 77843
(Dated: February 9, 2008)

The ground-state Gamow-Teller transition in the decay of ^{14}O is strongly hindered and the electron spectrum shape deviates markedly from the allowed shape. A reanalysis of the only available data on this spectrum changes the branching ratio assigned to this transition by seven standard deviations: our new result is (0.54 ± 0.02)%. The Kurie plot data from two earlier publications are also examined and a revision to their published branching ratios is recommended. The required nuclear matrix elements are calculated with the shell model and, for the first time, consistency is obtained between the M1 matrix element deduced from the analog gamma transition in ^{14}N and that deduced from the slope in the shape-correction function in the beta transition, a requirement of the conserved vector current hypothesis. This consistency is only obtained, however, if renormalized rather than free-nucleon operators are used in the shell-model calculations. In the mirror decay of ^{14}C a similar situation occurs. Consistency between the ^{14}C lifetime, the slope of the shape-correction function and the M1 matrix element from gamma decay can only be achieved with renormalized operators in the shell-model calculation.

PACS numbers: 23.40.Hc, 21.60.Cs, 21.10.Tg, 27.20.+n

I. INTRODUCTION

The nucleus ^{14}O decays predominantly by a Fermi transition to the 2.313 MeV 0^+ state in ^{14}N. Weak Gamow-Teller branches are evident to the 1^+ state at 3.95 MeV in ^{14}N (branching ratio, $R_{GT} = 0.055\%$) and to the 1^+ ground state with $R_{GT} \sim 0.6\%$. It can be concluded then that the Fermi transition has a branching ratio of $\sim 99.3\%$. Since systematic studies of Fermi superallowed transitions require their branching ratios be known to an accuracy of $\pm 0.1\%$, the branching ratio of the ground-state Gamow-Teller transition for ^{14}O must be determined to within 10% of its central value.

This Gamow-Teller transition is strongly inhibited. Its ft-value is roughly 10^4 times larger than is typical for favoured $0^+ \rightarrow 1^+$ transitions. (Even more inhibition is evident in the analog $^{14}\mathrm{C} \rightarrow {}^{14}\mathrm{N}$ transition.) The inhibition is attributed to accidental cancellation in the allowed Gamow-Teller matrix element for this transition [1]. Because the allowed matrix elements are so small, the induced terms (particularly "weak magnetism"), as well as the relativistic and the second-forbidden terms are expected to contribute appreciably to the decay probability. As a consequence, many of the usual assumptions in the allowed approximation may not be valid. For example, the spectrum shape may deviate markedly from the allowed (or statistical) spectrum shape.

To date there has only been one measurement, by Sidhu and Gerhart [2, 3], of the detailed shape of the beta spectrum from ^{14}O decay, from which they determined the branching ratio of the ground-state Gamow-Teller transition with the required precision. It was performed with an iron-free, beta-ray spectrometer, and was published 40 years ago! More recently, calculations by García and Brown [4] could not satisfactorily fit the observed beta spectrum, which led the authors to suggest there might be some systematic problem with Sidhu and Gerhart's measurement. This conclusion would have a serious impact on the branching ratio for the Fermi transition. For this reason, we have reanalyzed the data of Sidhu and Gerhart, which we obtained from a copy of Sidhu's Ph.D. thesis [3]. Our conclusion is that the ^{14}O spectrum shape can be understood, but only if renormalized operators are used in the shell-model calculations of the nuclear matrix elements. Our re-analysis yields a ground-state branching ratio of (0.54 ± 0.02)%, compared with the originally published result [2] of (0.61 ± 0.01)% – a large shift in terms of the uncertainties quoted.

II. RE-ANALYSIS OF SIDHU-GERHART EXPERIMENT

The probability per unit time for emission of a positron whose momentum lies between p and $p + dp$ is

$$\frac{dN}{dp} = \frac{G_{\mathrm{F}}^2 V_{ud}^2}{2\pi^3 (\hbar c)^6 \hbar} p^2 (E_0 - E)^2, \quad F(Z,E) S(Z,E) Q(p) R(p) \ \mathrm{s}^{-1}, \tag{1}$$

where G_{F} is the weak interaction coupling constant, V_{ud} the up-down matrix element of the CKM matrix, E the positron kinetic energy in MeV units, E_0 its maximum value, p the positron momentum, $F(Z,E)$ the Fermi function, Z the atomic number of the daughter nucleus, $S(Z,E)$ the shape-correction function, $Q(p)$ an atomic screening correction, and $R(p)$ a radiative correction. (The average value of $R(p)$, when integrated over an allowed electron spectrum, is denoted δ_R' in our publications [5] on superallowed Fermi transitions.) In this work, the shape-correction function includes the nuclear

*Present address: Department of Physics, Queen's University, Kingston, Ontario K7L 3N6, Canada

matrix elements[1]. We will write

$$\frac{2\pi^3(\hbar c)^6\hbar\ln 2}{G_{\rm F}^2 V_{ud}^2(m_e c^2)^5} = \overline{K} = 6146 \text{ s} \qquad (2)$$

and let all the instrumental factors be encompassed by some function $K(p)$. For an iron-free beta spectrometer, Sidhu assumes the instrumental function is proportional to the positron momentum, that is $K(p) = \kappa p$. If dN/dp is now interpreted as the number of counts in the beta-ray spectrometer per unit time that correspond to positrons whose momentum is between p and $p + dp$, we can write

$$\frac{dN}{dp} = Ap^3(E_0 - E)^2 F(Z,E)S(Z,E)Q(p)R(p), \qquad (3)$$

where $A = \kappa \ln 2/\overline{K}(m_e c)^5$ and is a constant of unknown magnitude. To calibrate the spectrometer and determine A, Sidhu made measurements of the lower-energy Fermi transition, for which all the factors on the right-hand-side of Eq. (3) are calculable, including the shape-correction function $S(Z,E)$. In Fig. 22 of his thesis, Sidhu showed the measured Kurie plot for the Fermi transition defined as

$$I(E) \equiv \sqrt{\frac{dN/dp}{p^3 F(Z,E)}} = A^{1/2}(E_0 - E) [S(Z,E)Q(p)R(p)]^{1/2}. \qquad (4)$$

In selecting an energy at which A is to be determined, Sidhu [3] writes "we want to avoid being too close to the end-point, because of the larger proportion of the high-energy positrons (from the Gamow-Teller decay) being mixed in this branch near the end point. On the other hand, we want to avoid positrons due to contaminants such as ^{15}O. The point at 1.691 MeV should be a good compromise." At $E = 1.691$ MeV, after reducing his result by 1.7% to eliminate the estimated contribution of ^{15}O contaminants, Sidhu quotes[2]

$$\frac{I^2(E)}{(E_0 - E)^2} = 1.232 \pm 0.013 = AS(Z,E)Q(p)R(p). \qquad (5)$$

Without the benefit of today's computing facilties, Sidhu assumed the product $S(Z,E)Q(p)R(p)$ to be equal to 2, the square of Fermi matrix element. We have now computed these factors more exactly and obtained: $S(Z,E) = 1.00226 \times 2.0$, $Q(p) = 1.00058$ and $R(p) =$ 0.99523. This product is 0.2% less than Sidhu's choice and yields

$$A = 0.6173 \pm 0.0064. \qquad (6)$$

[1] Further, these nuclear matrix elements include the coupling constants in their definition. For example, the Gamow-Teller matrix element, M_{GT}, includes the axial-vector coupling constant $g_{\rm A}$.

[2] Actually he quotes a value of 1.237±0.013, but a trivial numerical error on page 138 of the thesis indicates that one entry there is incorrectly recorded. We have worked backwards from the published result [2] to deduce what this value needs to be. Note the discrepancy is within the stated error.

TABLE I: Corrected shape-correction functions, $C(Z,W)$, obtained from Sidhu and Gerhart's data, $J(E)$, via Eq. (8).

E MeV	$J(E)$ Units:10^{-4}	F_0L_0/F	$Q(p)$	$R(p)$	$C(Z,W)$ Units:10^{-4}
2.235	1.738(10)	1.00296	1.00046	1.00907	2.781(16)
2.395	1.720(10)	1.00309	1.00043	1.00736	2.755(16)
2.544	1.669(10)	1.00321	1.00041	1.00571	2.679(16)
2.702	1.652(10)	1.00334	1.00039	1.00388	2.656(16)
2.860	1.616(10)	1.00347	1.00037	1.00195	2.603(16)
3.018	1.602(10)	1.00359	1.00035	0.99987	2.585(16)
3.172	1.553(10)	1.00372	1.00034	0.99767	2.512(16)
3.323	1.537(10)	1.00384	1.00033	0.99527	2.492(16)
3.482	1.515(10)	1.00397	1.00031	0.99239	2.463(16)
3.640	1.500(13)	1.00410	1.00030	0.98893	2.447(21)
3.798	1.480(17)	1.00423	1.00029	0.98443	2.424(28)

With A now determined, we consider the higher-energy lower-intensity Gamow-Teller transition. For this, we need to perform a nuclear-structure calculation of $S(Z,E)$ and compare with the experimental data. To this end it is convenient to introduce the shape-correction function defined by Behrens and Bühring [6]:

$$C(Z,W) = \frac{F(Z,E)}{F_0 L_0} S(Z,E), \qquad (7)$$

where W is the total positron energy in electron rest-mass units, $W = 1 + E/(m_e c^2)$. The factor F/F_0L_0 corrects for the fact that Behrens and Bühring compute the electron density at the origin and not at the nuclear radius R, as was historically the case. In Fig. 20 of his thesis, Sidhu plots his experimental data for the shape-correction function (corrected for backscattering) for eleven positron kinetic energies between 2.2 and 3.8 MeV. Two lower energy points are disregarded as being unreliable. We convert these data to the Behrens and Bühring-defined shape-correction function, and further apply a screening and radiative correction as follows:

$$C(Z,W) = \frac{J(E)}{AQ(p)R(p)} \frac{F(Z,E)}{F_0 L_0}. \qquad (8)$$

Here we have labelled Sidhu's original data as $J(E)$. In Table I we give $J(E)$ as well as the corrections F/F_0L_0, $Q(p)$, $R(p)$ and the determined values of $C(Z,W)$, which are also plotted in Fig. 1.

III. CALCULATION OF $C(Z,W)$

We will use the formalism of Behrens and Bühring [6] for the shape-correction function, where it is written in

terms of amplitudes $M_K(k_e,k_\nu)$ and $m_K(k_e,k_\nu)$:

$$C(Z,W) = \sum_{k_e k_\nu K} \lambda_{k_e} \left\{ M_K^2(k_e,k_\nu) + m_K^2(k_e,k_\nu) - \frac{2\mu_{k_e}\gamma_{k_e}}{k_e W} M_K(k_e,k_\nu) m_K(k_e,k_\nu) \right\}. \quad (9)$$

Here λ_{k_e} and μ_{k_e} are beta-decay Coulomb functions, which depend on the amplitudes of the electron wave functions at the origin, and are defined such that their values are of order unity, with corrections of order $(\alpha Z)^2$. These quantities have been tabulated by Behrens and Jänecke [7]. The factor γ_{k_e} is defined as $[k_e^2 - (\alpha Z)^2]^{1/2}$. Here k_e and k_ν are the partial-wave expansion labels for the electron and neutrino wave functions. In our evaluations we will keep the lowest two partial waves in each. Finally, K is the multipolarity of the transition operators. For the ground-state Gamow-Teller decay, $0^+ \to 1^+$, the multipolarity is restricted to $K = 1$ only.

Behrens and Bühring [6] give approximate expressions for $C(Z,W)$ by expanding the electron wave functions in a power series in WR and (αZ), and the neutrino wave function in a power series in $p_\nu R$, where p_ν is the neutrino momentum, and R is a typical nuclear size parameter. We have not followed this procedure, but rather we have computed Eq. (9) exactly by solving the Dirac equation for electron wave functions as described in Ref. [8]. Our only simplifying assumption is in the evaluation of $M_K(k_e,k_\nu)$ and $m_K(k_e,k_\nu)$, where the nuclear form factors of Behrens and Bühring are replaced in the 'impulse approximation' by nuclear reduced matrix elements. We compute these reduced matrix elements within the nuclear shell model.

For $A = 14$ nuclei, we use the $0p$-shell wave functions of Cohen and Kurath [9] denoted (8-16)POT. For comparison purposes we also consider the more recent $0p$-shell part of the Warburton-Brown Hamiltonian [10] (the interaction labelled PWBT in Table X of Ref. [10]). This interaction was determined from a least squares fit to 51 $0p$-shell binding energies for which the rms deviation of the fit was 378 keV.

These two sets of interactions only incorporate the $0p$-shell. We wanted also to examine the possible effects of sd-shell contributions. Close to major shell closures the choice of a model space and effective interaction can be problematic if one wants to go beyond simple single-major-shell configurations. For example, in the $A = 14$ spectrum the lowest-energy states are predominantly two holes outside a closed ^{16}O core, $|2h\rangle$, but lying low in the spectrum are 'intruder' states with configurations involving four holes and two particles, $|4h\text{-}2p\rangle$. Mixing between these configurations must occur, and to obtain the degree of mixing with the shell model is difficult. Shell-model calculations that attempt to mix $|2h\rangle$ and $|4h\text{-}2p\rangle$ configurations encounter what has been called [10] the "$n\hbar\omega$ catastrophe". The presence of $|4h\text{-}2p\rangle$ configurations depresses the $|2h\rangle$ states, opening up a large energy gap between the $|2h\rangle$ and $|4h\text{-}2p\rangle$ states. This would be corrected somewhat if the model calculation included $|6h\text{-}4p\rangle$ states as well, since the role of the $|6h\text{-}4p\rangle$ states is to depress the $|4h\text{-}2p\rangle$ states. Thus when truncating the model space to include only $|2h\rangle$ and $|4h\text{-}2p\rangle$ states, the depression driven by the $|6h\text{-}4p\rangle$ states on the $|4h\text{-}2p\rangle$ states is absent. In an attempt to circumvent this catastrophe we will use quite weak cross-shell interactions and examine the sensitivity of our results to the strength of the cross-shell interaction.

We adopt the following method for incorporating sd-shell effects in the mass-14 system: We use the Cohen-Kurath interaction [9] for p-shell interactions, the USD [11] for s,d-shell interactions and the Millener-Kurath [12] interaction for the cross-shell matrix elements. The Millener-Kurath interaction was designed to reproduce the unnatural-parity states in p-shell nuclei, such as the negative-parity states in $A = 14$ that involve just one particle in the s,d-shell. It wasn't designed to give the mixing between $|2h\rangle$ and $|4h\text{-}2p\rangle$ configurations. Nevertheless we will use the $\langle 2h|V|4h\text{-}2p\rangle$ matrix elements given by the Millener-Kurath interaction and multiply the matrix elements by a factor, f, that ranges from 0.0 to 0.6. When $f = 0.0$, there is no mixing between the $|2h\rangle$ and $|4h\text{-}2p\rangle$ configurations, and when $f = 0.6$ the ground-state wave function is approximately 74% $|2h\rangle$ and 26% $|4h\text{-}2p\rangle$. We will quote results using f=0.3, and denote this interaction as MK. We have examined the sensitivity of our results to a variation of f and found that the spread of the different results is within the assigned errors.

We are interested in further refining the wave function for the 1^+ $T = 0$ state in $A = 14$. As was noted by García and Brown [4], who suggested this procedure, the GT transition strength to the lowest 1^+ state is very small compared to the strength of the transition to the second 1^+ state at 3.95 MeV excitation energy. Thus, any small mixing between these two 1^+ states in the model will have a large effect on the weak transition rate. To make use of this fact, we can write the wave function for the lowest 1^+ state in ^{14}N as

$$|1^+ \text{ low}\rangle = \alpha|1^+(1)\rangle + \beta|1^+(2)\rangle \quad (10)$$

with $\alpha^2 + \beta^2 = 1$. Here (1) and (2) refer to the first and second model states obtained with either the CKPOT, PWBT or MK effective Hamiltonians. In fitting the beta-decay data, it turns out that we need a negative sign for β with the CKPOT interaction and a positive sign with the PWBT and MK interactions.

Our strategy is then to adjust α to minimize the χ^2 between the calculated $C(Z,W)$ and the corrected experimental shape-correction function given in Table I. The Gamow-Teller matrix element, M_{GT}, is particularly sensitive to small variations in α, and consequently is rather precisely determined mainly from the fit to the absolute magnitude to the shape-correction function. We have fitted our calculated $C(Z,W)$ to the expression

$$C(Z,W) = |M_{GT}|^2 k(1 + aW + \mu_1\gamma_1 b/W + cW^2) \quad (11)$$

TABLE II: The value of the wave function amplitude, α, Eq. (10) that gave the best fit to the experimental shape-correction function for ^{14}O, and the best fit to the decay half-life for ^{14}C. Also given are the Gamow-Teller and $M1$ matrix elements for the $0^+ \to 1^+$ transition, the parameters k and a of the shape-correction function, $C(Z,W)$, and the ground-state branching ratio, R_{GT}. The χ^2 per degree of freedom, χ^2/ν for the fit to the ^{14}O shape-correction function is recorded. Free-nucleon operators were used for the shell-model calculations.

Model	α	χ^2/ν	M_{GT}	M^{β}_{M1}	k	a MeV^{-1}	R_{GT}(%)
^{14}**O decay:**							
CK	0.98447	2.0	0.01400	−0.533	1.999	−0.133	0.566
PWBT	0.99805	3.5	0.01392	−0.569	2.068	−0.139	0.570
MK	0.98482	1.5	0.01406	−0.515	1.958	−0.130	0.563
^{14}**C decay:**							
CK	0.98560		−0.00465	0.503	0.345	−0.647	
PWBT	0.99760		−0.00469	0.539	0.346	−0.683	
MK	0.98595		−0.00458	0.484	0.354	−0.624	
Expt: $\lvert M^{\beta}_{M1}\rvert$ from Γ_γ in ^{14}N				0.312(7)			
Expt: ^{14}C slope a						−0.45(4)	

as the approximate expressions of Behrens and Bühring can be cast in this form. In Table II we give the first two parameters, k and a, as determined by least-squares fitting for each set of shell-model interactions. The parameter a, which is called the slope of the shape-correction function, can be expressed in approximate form [4, 6] as[3]

$$a_{\rm approx} = \frac{8}{3M}\frac{M^{\beta}_{M1}}{M_{GT}}, \qquad (12)$$

where M is the nucleon mass and M^{β}_{M1} is the $M1$ matrix element. Clearly this slope is dominated by an interference between the Gamow-Teller and $M1$ matrix elements. This is of considerable – indeed historical – importance [13]. A measurement of M^{β}_{M1} obtained from beta decay can be compared with the corresponding matrix element, M^{γ}_{M1}, obtained from the electromagnetic transition between the isobaric analog state and the ground state of ^{14}N. The conserved vector current hypothesis as enunciated by Gell-Mann [13] proposes that the vector current of the weak interaction is just a rotation in isospin of the vector current of the electromagnetic interaction; in this case we would expect $M^{\beta}_{M1} = -M^{\gamma}_{M1}/\sqrt{2}$. (The $\sqrt{2}$ factor originates in the isospin Clebsch-Gordan coefficient for a charge-changing reaction, such as beta-decay, differing by $\sqrt{2}$ from the isospin Clebsch-Gordan coefficient for a charge-conserving reaction, such as gamma-decay.) The analogous electromagnetic transition in ^{14}N is the 0^+ $T=1 \to 1^+$ $T=0$ decay of the 2.313 MeV state, which has a measured gamma width [14] of $\Gamma_\gamma = (6.7 \pm 0.3) \times 10^{-9}$ MeV. The corresponding M1 matrix element is

$$|M^{\gamma}_{M1}|_{\rm expt} = \sqrt{\frac{3\Gamma_\gamma}{4(E_\gamma/\hbar c)^3 \mu_N^2}} = 0.442 \pm 0.010. \qquad (13)$$

Here E_γ is the photon energy in MeV, $E_\gamma = 2.313$ MeV, and μ_N the nuclear magneton unit, $\mu_N = 0.1262$ MeV$^{1/2}$ fm$^{3/2}$. The matrix element is dimensionless. To conform with CVC, the beta-decay $M1$ matrix element should be given by

$$|M^{\beta}_{M1}| = |M^{\gamma}_{M1}|/\sqrt{2} = 0.312 \pm 0.007, \qquad (14)$$

if we neglect charge-symmetry breaking corrections. García and Brown [4] have studied this issue and concluded that "one should not expect very large charge-symmetry-breaking effects in the $A=14$ system".

It is evident in Table II that for values of α that give a good fit to the experimental beta-decay spectrum shape the value of the $M1$ matrix element differs from Eq. (14) by about a factor of 1.7. This disappointing result is also consistent with the conclusions of García and Brown [4]: with $0p$-shell wave functions it is not possible to fit the $B(M1; 0^+ \to 1^+)$ radiative decay simultaneously with the beta-decay measurements of Sidhu and Gerhart [2, 3].

We have also examined the mirror decay of ^{14}C, whose lifetime is known [15] to be $t_{1/2} = 5700(30)$ yr. Our strategy for this decay is to adjust the wavefunction amplitude

[3] This approximation is quite poor. We only display it to show the important effect of the $M1$ matrix element.

TABLE III: Same as Table II, except that renormalized operators are used rather than free-nucleon operators in the shell-model calculations.

Model	α	χ^2/ν	M_{GT}	M^{β}_{M1}	k	a MeV^{-1}	$R_{GT}(\%)$
^{14}**O decay:**							
CK	0.98428	5.1	0.01490	−0.313	1.552	−0.099	0.540
PWBT	0.99812	3.3	0.01480	−0.346	1.605	−0.105	0.544
MK	0.98463	6.0	0.01494	−0.300	1.526	−0.095	0.538
^{14}**C decay:**							
CK	0.98556		−0.00338	0.281	0.613	−0.421	
PWBT	0.99762		−0.00343	0.314	0.607	−0.459	
MK	0.98590		−0.00333	0.267	0.629	−0.406	
Expt: $\lvert M^{\beta}_{M1}\rvert$ from Γ_γ in ^{14}N				0.312(7)			
Expt: ^{14}C slope a						−0.45(4)	

α to fit this lifetime exactly. Then the shell-model calculations give a prediction for the shape-correction function, $C(Z,W)$, whose slope parameter a can be compared with the experimentally determined value of Wietfeldt *et al.* [16]. The results are listed in the bottom half of Table II. Again we have a disappointing result: the calculated slope parameter of $a \simeq -0.65$ MeV^{-1} differs significantly from the experimental value[4] of $-0.45(4)$ MeV^{-1}.

A. Renormalized operators

We carried out the shell-model calculations just discussed using operators derived in the impulse approximation with coupling constants appropriate for free nucleons. In finite nuclei one expects corrections to this scheme coming from two sources: firstly, the shell-model calculation is carried out in a truncated model space, which can be corrected in a perturbation expansion, and secondly the nucleons in the nucleus are interacting via the exchange of mesons and these mesons can influence the electromagnetic and weak interactions in nuclei. The two corrections are called core polarization and meson-exchange currents respectively. These phenomena are responsible for the quenching of the Gamow-Teller matrix element in finite nuclei. We define

$$g_{\rm A,eff} = g_{\rm A} + \delta g_{\rm A}, \tag{15}$$

where $g_{\rm A}$ is the free-nucleon value of the axial-vector coupling constant, $g_{\rm A} \simeq 1.27$, and $\delta g_{\rm A}$ the correction to it. We fix the value of $\delta g_{\rm A}$ by considering the beta decay of ^{15}O to its ground-state mirror in ^{15}N, whose experimentally determined Gamow-Teller matrix element [17] shows a reduction of 13.2(7)% over that calculated with the free-nucleon coupling constant for a configuration that consists of a single $0p$-hole in a closed ^{16}O core. To fit the experimental value requires

$$\delta g_{\rm A} = -0.165, \tag{16}$$

and we will adopt this value.

We also need to understand how the isovector $M1$ operator is renormalized by core polarization and meson-exchange currents. Both these corrections have been evaluated in an *ab initio* calculation for closed-shell-plus-or-minus-one nucleon configuration by Towner and Khanna [18]. Their results for the $A = 15$ case of a $0p$ hole in an ^{16}O core are expressed in terms of an effective $M1$ operator defined as

$$(M1)_{\rm eff} = g^{(1)}_{\rm L,eff}\mathbf{L} + g^{(1)}_{\rm S,eff}\mathbf{S} + g^{(1)}_{\rm P,eff}[Y_2,\mathbf{S}], \tag{17}$$

where $g^{(1)}_{\rm X,eff} = g^{(1)}_{\rm X} + \delta g^{(1)}_{\rm X}$, with $g_{\rm X}$ being the free-nucleon coupling constant, $\delta g_{\rm X}$ the calculated correction to it and $X = L$, S or P. The free-nucleon values are $g^{(1)}_{\rm L} = 0.5$, $g^{(1)}_{\rm S} = 4.706$ and $g^{(1)}_{\rm P} = 0$. Towner and Khanna calculated the isovector combinations of the magnetic moments of the ^{15}O and ^{15}N ground states and obtained an 8.9% enhancement in the free-nucleon isovector magnetic moment compared to an experimental enhancement of 11.1% – a clear success for an *ab initio* calculation. We, therefore, adopt their calculated values of

$$\delta g^{(1)}_{\rm L} = 0.076 \qquad \delta g^{(1)}_{\rm S} = -0.22 \qquad \delta g^{(1)}_{\rm P} = 0.96. \tag{18}$$

[4] We take the slope parameter from Table II of [16] using the fitted value for the electron spectrum in the 100 to 160 keV energy range. In a wider energy range, 50 – 160 keV, the authors quote a slope parameter of -0.32 MeV^{-1}.

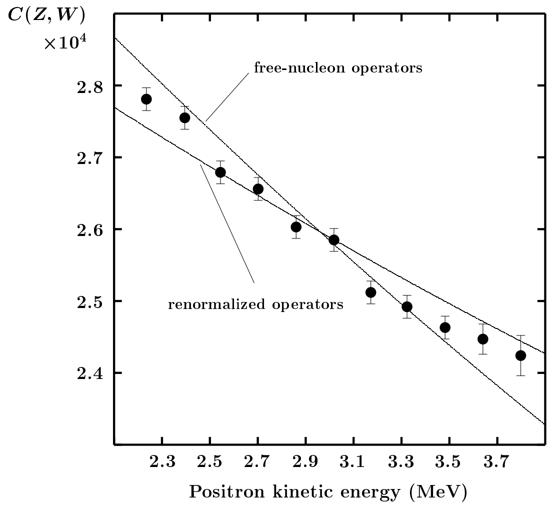

FIG. 1: The shape-correction function calculated with PWBT wave functions compared with the experimental data of Sidhu and Gerhart [3].

TABLE IV: Statistical rate function f, Eq. (20), for the Gamow-Teller transitions in ^{14}O and ^{14}C decays.

	f	
Model	^{14}O	^{14}C
free-nucleon operators		
CK	2473.1	1.5552×10^{-3}
PWBT	2522.9	1.5284×10^{-3}
MK	2441.8	1.6040×10^{-3}
renormalized operators		
CK	2086.4	2.9314×10^{-3}
PWBT	2128.1	2.8437×10^{-3}
MK	2067.5	3.0222×10^{-3}
$f_{\rm stat}$	1633.6	6.0766×10^{-3}

With the effective operators thus selected, we repeated the strategy of adjusting the wave function amplitude α to minimize the χ^2 between the calculated $C(Z,W)$ and the corrected experimental shape-correction function given in Table I. The results of the fit are given in Table III, where it is observed that the M^{β}_{M1} matrix element is considerably reduced from that in Table II and is now comparable to that deduced from gamma decay. This is a major success and a strong endorsement for the use of renormalized operators. However, it is not all good news. For two of the wave function choices, CK and MK, the quality of the fit to the experimental shape-correction function is inferior, the χ^2 per degree of freedom being two to four times larger. Only for the PWBT interaction is the quality of the fit comparable. We show the PWBT fit in Fig. 1. It is clear that the renormalized operators lead to a smaller slope in the shape-correction function. This is the expected result since the slope is governed by the matrix-element ratio M^{β}_{M1}/M_{GT}, as given in Eq. (12).

For the mirror decay in ^{14}C, our strategy was to fix the wavefunction amplitude α to reproduce the known lifetime, and use these wave functions to compute the shape-correction function $C(Z,W)$, giving a prediction for the slope parameter, a. From Table III it is clear that the use of renormalized operators is very successful: the calculated slope parameter of $a \simeq -0.43$ MeV^{-1} agrees perfectly with the experimental result of Ref. [16].

IV. THE ^{14}O GAMOW-TELLER BRANCHING RATIO

Now that a shape-correction function $C(Z,W)$ has been obtained for ^{14}O decay that agrees reasonably well with the data of Sidhu and Gerhart [2, 3] (see Fig. 1) we integrate this function over the entire beta spectrum and compare with the analogous Fermi transition to obtain the Gamow-Teller branching ratio, R_{GT}:

$$\begin{aligned} R &= \frac{t_F}{t_{GT}} = \frac{N_{GT}}{N_F} = \frac{f_{GT}(1+\delta_R')_{GT}|M_{GT}|^2}{f_F(1+\delta_R')_F|M_F|^2(1-\delta_C)}, \\ R_{GT} &= \frac{R}{1+R}, \end{aligned} \tag{19}$$

where t_F and t_{GT} are the partial half-lives of the Fermi and Gamow-Teller transitions respectively, and N_F and N_{GT} are their integrated count rates, $N = \int (dN/dp)dp$. For the Gamow-Teller transition, the statistical rate function is defined as

$$f = \frac{1}{|M_{GT}|^2} \int_1^{W_0} pW(W_0-W)^2 F_0 L_0 C(Z,W) Q(p) dW. \tag{20}$$

Note that, since $C(Z,W)$ includes the nuclear matrix elements, we have divided by $|M_{GT}|^2$ to conform with the normal definition of f. The calculated values for the Gamow-Teller transition are listed in Table IV for the various wave function selections.

The other factors in Eq. (19) were evaluated as follows: For the Fermi transition we calculate $f_F = 42.772$ (see Ref. [8]) and $|M_F|^2 = 2.0$. Next, using the methods described in Ref. [5] we obtain $(\delta_R')_{GT} = 1.294\%$, $(\delta_R')_F = 1.520\%$ and $\delta_C = 0.57\%$. With these values we obtain the branching ratios listed in the last column of Tables II and III. We adopt the result with renormalized operators as our central value and assign an error that is half the spread between the renormalized and free-nucleon operators, obtaining $R_{GT} = (0.540 \pm 0.015)\%$. However, there still remains a 1% normalization uncertainty in the value of the calibration constant A, Eq. (6).

We add this uncertainty linearly to obtain as our final branching ratio a value of

$$R_{GT} = (0.54 \pm 0.02)\%. \tag{21}$$

This result differs from Sidhu and Gerhart's [2] published result of $R_{GT} = (0.61 \pm 0.01)\%$ and has a larger uncertainty. The Gamow-Teller matrix element from the fit is $M_{GT} = 0.0149 \pm 0.0005$ compared to Sidhu and Gerhart's published result of $M_{GT} = 0.0164 \pm 0.0004$.

V. IMPACT ON SUPERALLOWED BRANCH

A. Two earlier measurements

There are two earlier measurements of the ^{14}O branching ratio: Sherr *et al.* [19] obtained $R_{GT} = (0.6 \pm 0.1)\%$, while Frick *et al.* [20] obtained $(0.65 \pm 0.05)\%$. In both cases Kurie plots were constructed from raw data for both the Fermi and Gamow-Teller transitions and the required branching ratio was obtained from their ratio:

$$R = \frac{f_{GT}}{f_F}\frac{X_{GT}^2}{X_F^2} \qquad R_{GT} = \frac{R}{1+R}, \tag{22}$$

where X_F, X_{GT} are the ratio of the Kurie plot data to the allowed approximation $(W_0 - W)$, *viz.*

$$X = \frac{1}{n_i}\sum_{i=1}^{n_i}\frac{K(W_i)}{W_0 - W_i} \qquad K(W) = \sqrt{\frac{dN/dp}{p^2 F(Z,W)}}. \tag{23}$$

Here W_i are the values of W for which experimental data have been obtained, and n_i is the number of such data. The unknown normalization of the Kurie plots cancels in the ratio. Further, f is the integrated electron spectrum, which in the allowed approximation is

$$f_{\rm stat} = \int_1^{W_0} pW(W_0 - W)^2 F(Z,W) Q(p) dW. \tag{24}$$

Sherr *et al.* [19] only published their Gamow-Teller Kurie plot, so it is not possible to reanalyze their result. Frick *et al.* [20], on the other hand, published both their Fermi and Gamow-Teller Kurie plots, so we have reanalyzed them according to Eq. (22) obtaining $R_{GT} = (0.64 \pm 0.03)\%$ agreeing satisfactorily with the published result. (Our uncertainty only includes the statistical uncertainty in the fit of the allowed shape to the Kurie data, and the uncertainty in the end point energy, W_0, as known in 1963).

The original analyses of both experiments were based on the allowed approximation. However, as we have discussed at length, the ^{14}O Gamow-Teller transition is strongly hindered and the allowed approximation not sufficient. Thus, we have reanalyzed the Frick *et al.* [20] Kurie plot data using for X

$$X = \frac{1}{n_i}\sum_{i=1}^{n_i}\frac{K(W_i) M_{GT}}{(W_0 - W_i)\sqrt{C(Z,W_i)}}, \tag{25}$$

and using for the integrated spectrum, f, the exact expression given in Eq. (20). The shape-correction function, $C(Z,W)$, is evaluated with p-shell wave functions, PWBT, and renormalized operators. The result is an increase in the branching ratio to $R_{GT} = (0.73 \pm 0.03)\%$. We checked our procedure by performing the same analysis on Sidhu's Kurie plots, as published in his thesis [3]. In the allowed approximation we obtained $R_{GT} = (0.45 \pm 0.01)\%$, while with the exact expressions we obtained $R_{GT} = (0.53 \pm 0.01)\%$ in agreement with our more accurate analysis of the shape-correction functions. The conclusion is clear. For the Gamow-Teller branching ratio in ^{14}O, determinations based on an allowed approximation analysis of Kurie plots have to be increased by about 14%. Unfortunately this places the earlier data in conflict with the more accurate Sidhu-Gerhart result [2, 3]. This is not a question of the method of analysis: the raw data are in conflict. If one considers a ratio of ratios, comparing the ratio of Gamow-Teller to Fermi Kurie plots for Sidhu [3] and Frick *et al.* [20], discrepacies of order 20% are evident. This, alone, leads to a 40% difference in the deduced branching ratios. If a modern day experiment were to be mounted, this would be one discrepancy that could quickly be resolved.

B. The Fermi branch and corrected $\mathcal{F}t$ value

A survey of all the data on superallowed $0^+ \to 0^+$ Fermi decay has recently been published by Hardy and Towner [8]. For the ^{14}O Fermi branching ratio, the value of 99.334(10)% is given there based on the ground-state Gamow-Teller branching ratio obtained from Sidhu and Gerhart [2] averaged with the two older and less precise results from [19] and [20], and a second Gamow-Teller branching ratio to the 3.95 MeV state of 0.0545(19)%. If we now replace the Sidhu-Gerhart value with the result from Eq. (21), and increase the branching for the two older data by 14% as discussed in Sect. V A but leave their error assignments at their published values, then we obtain a very conservative estimate of the ground-state Gamow-Teller branching ratio of $(0.57 \pm 0.06)\%$. The uncertainty here has been scaled by 2.6 acoording to our usual prescription [8] because of the incompatility of the Sidhu-Gerhart result with the two older measurements. The Fermi branching ratio is now increased to 99.376(65)%. The impact of this is to lower the corrected $\mathcal{F}t$ value from 3071.9(26)s to 3070.7(32)s. This value still leaves the ^{14}O datum consistent with the other eleven precision measured $\mathcal{F}t$ values, although it is now on the low side of the average. The slight shift in ^{14}O is well within the stated errors in the survey [8] and has a negligible impact on the physics conclusions obtained there.

VI. A NOTE ON ft VALUES FOR GAMOW-TELLER TRANSITIONS

It is traditional to characterize an allowed Gamow-Teller transition by its $\log ft$ value, where the statistical rate function f used in this application is devoid of any nuclear-structure factors and is defined in Eq. (24). When the Gamow-Teller matrix element is large, of order unity, then the shape-correction function $C(Z,W)$ is nearly independent of energy and $C(Z,W)/|M_{GT}|^2$ is close to unity. Under these conditions there is little difference between the exactly defined f of Eq. (20) and the traditional expression in Eq. (24). But for the Gamow-Teller transitions in ^{14}O and ^{14}C decays, $|M_{GT}|$ is very small and the shape-correction function has a significant effect. For these transitions there is a large difference between the exact f and $f_{\rm stat}$ as shown in Table IV.

For the exactly-defined f, the transition ft-value equals a constant divided by the square of the Gamow-Teller matrix element

$$ft = \frac{2\pi^3 \ln 2 (\hbar c)^6 \hbar}{G_{\rm F}^2 V_{ud}^2 (m_e c^2)^5} \frac{1}{|M_{GT}|^2} = \frac{6146}{|M_{GT}|^2} \text{ s.} \qquad (26)$$

Note that in our notation $|M_{GT}|$ includes the axial-vector coupling constant, $g_{\rm A}$. For precision work, the lifetime t should be adjusted for radiative corrections. Eq. (26) is frequently used to deduce the Gamow-Teller matrix element from a published ft value for which $f_{\rm stat}$ has been used for f. For example, the National Nuclear Data Center [15] gives the $\log ft$ value for ^{14}C decay as 9.040(3) and for ^{14}O decay as 7.279(8). To deduce $|M_{GT}|$ from Eq. (26) with these values would be incorrect. The $\log ft$ values for the exactly defined f are 8.72(2) and 7.44(1) respectively, with larger error bars because of the uncertainty from nuclear structure.

VII. CONCLUSIONS

We began this work disturbed by the statement from García and Brown [4] that it was not possible to fit the $B(M1; 0^+ \to 1^+)$ radiative decay in ^{14}N simultaneously with the ^{14}O beta decay measurements of Sidhu and Gerhart [2], an apparent violation of the conserved vector current hypothesis. A problem with the Sidhu-Gerhart work "could significantly change the conclusions extracted from $0^+ \to 0^+$ transitions regarding universality and unitarity", they wrote. We initially reanalyzed the Sidhu-Gerhart experiment and came to the same conclusion as García and Brown, but we then discovered that by using renormalized operators in the shell-model calculation we could achieve much greater consistency between the requirements of CVC and the measurements of Sidhu and Gerhart. This observation was reinforced when we examined the mirror ^{14}C decay. There it was only possible to fit the known lifetime and the slope parameter in the shape-correction function [16] while remaining consistent with the requirements of CVC, if renormalized operators were used. This is our principal physics conclusion: the use of renormalized operators is mandatory.

A second important outcome relates to the superallowed beta decay of ^{14}O. With the reanalysis of the Sidhu-Gerhart experiment the recommended value from this experiment for the Gamow-Teller branching ratio from ^{14}O to the ground state of ^{14}N is $R_{GT} = (0.54 \pm 0.02)\%$ compared to the published value [2] of $(0.61 \pm 0.01)\%$. This result, when combined with updated older measurements, revises the recommended branching ratio for the Fermi transition from 99.334(10)% to 99.376(65)% and shifts the corrected $\mathcal{F}t$ value for the Fermi branch downwards by 1.2s. This shift is within the stated uncertainty of the $\mathcal{F}t$ value given in the survey of Hardy and Towner [8] and does not alter any of the conclusions reached there.

[1] H.J. Rose, O. Häusser and E.K. Warburton, Rev. Mod. Phys., **40**, 591 (1968).

[2] G.S. Sidhu and J.B. Gerhart, Phys. Rev. **148**, 1024 (1966).

[3] G.S. Sidhu, *An experimental study of the spectrum shape for the Gamow-Teller transition $^{14}O \to ^{14}N$*, Ph.D. thesis, University of Washington (1966).

[4] A. García and B.A. Brown, Phys. Rev. C **52**, 3416 (1995).

[5] I.S. Towner and J.C. Hardy, Phys. Rev. C **66**, 035501 (2002).

[6] H. Behrens and W. Bühring, *Electron Radial Wave Functions and Nuclear Beta-decay* (Clarendon Press, Oxford, 1982).

[7] H. Behrens and J. Jänecke, *Numerical Tables for Beta Decay and Electron Capture, Landolt-Börnstein, New Series I/4* (Springer-Verlag, Berlin, 1969).

[8] J.C. Hardy and I.S. Towner, Phys. Rev. C **71**, 055501 (2005).

[9] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).

[10] E.K. Warburton and B.A. Brown, Phys. Rev. C **46**, 923 (1992).

[11] B.H. Wildenthal, in *Progress in Particle and Nuclear Physics*, edited by D.H. Wilkinson (Pergamon Press, Oxford 1984) Vol. **11**, p. 5.

[12] D.J. Millener and D. Kurath, Nucl. Phys. **A255**, 315 (1975).

[13] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).
[14] F. Ajzenberg-Selove, Nucl. Phys. **A523**, 1 (1991).
[15] Average value recommended by the National Nuclear Data Center: `www.nndc.bnl.gov`
[16] F.E. Wietfeldt, E.B. Norman, Y.D. Chan, M.T.F. da Cruz, A. García, E.E. Haller, W.L. Hansen, M.M. Hindi, R.-M. Larimer, K.T. Lesko, P.N. Luke, R.G. Stokstad, B. Sur and I. Žlimen, Phys. Rev. C**52**, 1028 (1995).
[17] S. Raman, C.A. Houser, T.A. Walkiewicz and I.S. Towner, At. Data Nucl. Data Tables **21**, 567 (1978).
[18] I.S. Towner and F.C. Khanna, Nucl. Phys. **A399**, 334 (1983).
[19] R. Sherr, J.B. Gerhart, H. Horie and W.F. Hornyak, Phys. Rev. **100**, 945 (1955).
[20] G. Frick, A. Gallmann, D.E. Alburger, D.H. Wilkinson and J.P. Coffin, Phys. Rev. **132**, 2169 (1963).